

The effect of the warping term on the fractional quantum Hall states in topological insulators

Fawei Zheng,¹ Zhigang Wang,¹ Zhen-Guo Fu,² and Ping Zhang^{1,2,*}

¹*LCP, Institute of Applied Physics and Computational Mathematics,
P. O. Box 8009, Beijing 100088, China*

²*Beijing Computational Science Research Center, Beijing 100084, China*

Abstract

The warping effect on the fractional quantum Hall (FQH) states in topological insulators is studied theoretically. Based on the perturbed wavefunctions, which include contributions from the warping term, analytical expressions for Haldane's pseudopotentials are obtained. We show that the warping term does not break the symmetry of the pseudopotentials for $n=\pm 1$ Landau levels (LLs). With increasing the warping strength of the Fermi surface, our results indicate that the stability of the FQH states for LL $n = 0$ (LLs $n=\pm 1$) becomes stronger (weaker), and the excitation gap at $\nu = 1/3$ FQH state for LL $n=0$ also increases while the gaps for LLs $n=\pm 1$ are unchanged.

PACS numbers: 73.43.Lp, 73.20.At, 73.25.+i

*Corresponding author. Email address: zhang_ping@iapcm.ac.cn

Topological insulators (TIs) as a new phase of quantum matter, which can not be adiabatically connected to conventional insulators and semiconductors, have been intensively studied in recent years [1–10]. TIs are characterized by a full insulating gap in the bulk and protected gapless edge or surface states. Near the Fermi level the low-energy dispersion of the TI surface states shows a Dirac linear behavior. However, the recent angle-resolved photoemission spectroscopy experiments show that the Fermi surface in Bi_2Te_3 [9, 10], a typical TI, is a snowflake shape rather than a circle one. The origin of this snowflake-like Fermi surface has been confirmed to arise from an unconventional hexagonal warping term [11]. It is this warping term that brings about many unique physical phenomena [12–14], which can not be observed in other systems, including the extensively studied graphene and the conventional two-dimensional electron gas.

More recently, there has been emerging attention to the interactions of the Dirac-type quasiparticles and their strong correlation effects in TI, especially the TI fractional quantum Hall (TIFQH) states. Despite no undeniable experimental observation of the TIFQH states heretofore, some theoretical studies have been undertaken. For example, DaSilva [15] predicted the stability of the TIFQH states for Landau levels (LLs) with index $n=0$ and ± 1 in TIs. Apalkov and Chakraborty studied the finite thickness effect on the TIFQH states [16]. Also, the present authors investigated the influences of the Zeeman splitting and the tilted strong magnetic field on the stability of the TIFQH states [17, 18] with large g factor. However, many important and interesting open questions, such as the warping effect, the spin excitations, and the subband-LL coupling, have not been discussed in the TIFQH regime.

In this paper we theoretically study the warping effect on the TIFQH states. Here the warping term is perturbatively treated. With the help of the numerical calculations, we show that the warping term can not break the symmetry of the Haldane’s pseudopotentials for $n=\pm 1$ LLs, which differs from the role of the spin splitting [17]. Moreover, our results indicate that with the increase of the warping strength of the Fermi surface, the stability of the TIFQH states for LL $n = 0$ (LLs $n=\pm 1$) become stronger (weaker), and the excitation gap at $\nu = 1/3$ filling for LL $n=0$ (gaps for LLs $n=\pm 1$) also increases (keep unchanged).

In the presence of a perpendicular magnetic field $\mathbf{B} = B\hat{z}$, the effective Dirac Hamiltonian

for $\text{Bi}_2\text{Te}_3(111)$ surface is written as

$$H_0 = v_f (\sigma_x \Pi_y - \sigma_y \Pi_x) + \frac{\lambda}{2} (\Pi_+^3 - \Pi_-^3) \sigma_z, \quad (1)$$

where $\mathbf{\Pi} = \mathbf{k} + e\mathbf{A}/c$ with the wave vector $\mathbf{k} = (k_x, k_y, 0)$ and the gauge $\mathbf{A} = B(-y/2, x/2, 0)$. Here, $\Pi_{\pm} = \Pi_x \pm \Pi_y$, $\sigma_{x,y,z}$ are Pauli matrices, v_f denotes the Fermi velocity, and λ describes the hexagonal warping strength of the Fermi surface [11]. Here we have assumed that the Zeeman splitting is much weaker than the warping term and therefore can be neglected for the first step in order to solely illustrate the role played by the warping term. This is the case for $\text{Bi}_2\text{Te}_3(111)$ system. By introducing the ladder operators $a^\dagger = \frac{1}{\sqrt{2}} \left(\frac{z}{2l_B} - 2l_B \partial_{\bar{z}} \right)$ and $a = \frac{1}{\sqrt{2}} \left(\frac{\bar{z}}{2l_B} + 2l_B \partial_z \right)$ with $z(\bar{z}) = x \pm iy$ and the magnetic length $l_B = \sqrt{\hbar c / eB}$, we can rewrite the Hamiltonian (1) as

$$H_0 = \frac{\sqrt{2}}{l_B} \begin{pmatrix} -i(a^{\dagger 3} - a^3) \lambda / l_B^2 & v_f a \\ v_f a^\dagger & i(a^{\dagger 3} - a^3) \lambda / l_B^2 \end{pmatrix}. \quad (2)$$

When $\lambda=0$ the Hamiltonian (2) can be exactly solved, and the eigenstates are given by

$$\Psi_{n,m}^{(0)} = \begin{cases} \frac{1}{\sqrt{2}} \begin{pmatrix} \text{sgn}(n) |n| - 1, m \rangle \\ |n|, m \rangle \end{pmatrix}, & \text{for } n \neq 0, \\ \begin{pmatrix} 0 \\ |0, m \rangle \end{pmatrix}, & \text{for } n = 0, \end{cases} \quad (3)$$

where the symbol $|n, m\rangle$ is the non-relativistic two-dimensional electron gas Landau eigenstates with non-relativistic quadratic dispersion relation in n th LL with angular momentum m . The corresponding LLs are expressed as $\varepsilon_n = \text{sgn}(n) v_f \sqrt{2|n|} / l_B$. When the warping term is taken into account ($\lambda \neq 0$), however, the single-particle eigenstate can not be obtained directly. Fortunately, one can take the perturbation method to get eigenstates of Hamiltonian (2) since the warping term, $\sqrt{2}\lambda l_B^{-3}$, is much smaller than the typical energy space between the nearest-neighboring LLs, $\sqrt{2}v_f / l_B$, i.e., $\zeta \equiv \lambda / (v_f l_B^2) \ll 1$. After some long but straightforward algebraic operations, and only keeping the first-order terms, we have

$$\Psi_{n,m} = \Psi_{n,m}^{(0)} + \begin{pmatrix} \chi_{|n|+2}^{(1)} |n| + 2, m \rangle \\ \chi_{|n|-3}^{(2)} |n| - 3, m \rangle \end{pmatrix}, \quad (4)$$

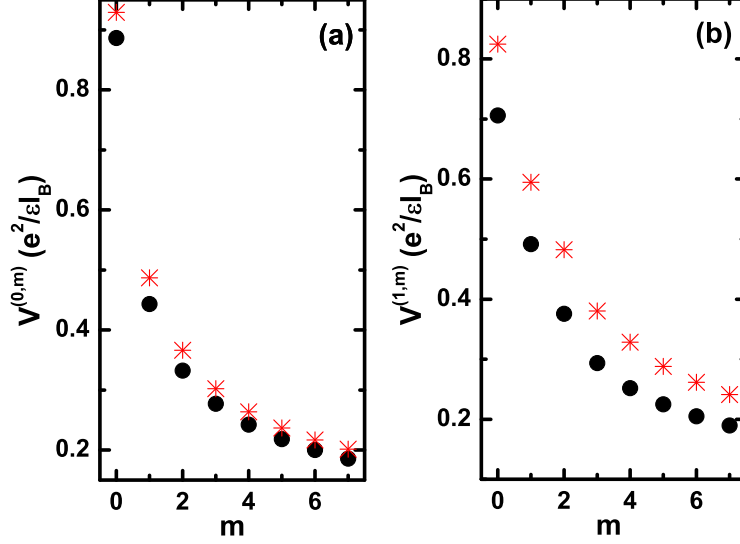


FIG. 1: (Color online) The effective pseudopotentials of the Coulomb interaction $V^{(n,m)}$ between two electrons at (a) $n=0$ and (b) $n=1$ LLs as functions of the relative angular momentum with different warping strength $\zeta=0$ (circles) and $\zeta=0.2$ (stars).

where the coefficients are

$$\chi_{|n|+2}^{(1)} = -i\zeta \sqrt{\frac{(|n|+1)(|n|+2)}{2}},$$

$$\chi_{|n|-3}^{(2)} = \begin{cases} -i\text{sgn}(n)\zeta \sqrt{\frac{(|n|-1)(|n|-2)}{2}} & |n| > 2 \\ 0 & |n| \leq 2 \end{cases}. \quad (5)$$

In the following discussion we will focus our attention to the TIFQH states of $|n| \leq 1$, because the stable TIFQH states can only be observed for LLs $|n| \leq 1$ [15]. The Haldane's pseudopotential [19] for Coulomb interaction $V(\mathbf{r}) = \frac{e^2}{\epsilon r}$ between electrons in the n th LL with relative angular momentum m is given by

$$V^{(n,m)} = \sum_{\mathbf{q}} \frac{\pi e^2}{\epsilon q} [\mathcal{F}(q)]^2 L_m(q^2 l_B^2) e^{-\frac{q^2 l_B^2}{2}}, \quad (6)$$

in terms of Laguerre polynomials $L_m(x)$ and the form factor $\mathcal{F}(q) = \langle \Psi_n | e^{-i\mathbf{q}\cdot\boldsymbol{\eta}} | \Psi_n \rangle$ with the cyclotron variable $\boldsymbol{\eta} = \mathbf{r} - \mathbf{R}$. Here, \mathbf{R} is the guiding-center position. Explicitly, for LLs $|n| \leq 1$ we have

$$V^{(0,m)} = \sum_{\mathbf{q}} \frac{\pi e^2}{\epsilon q} L_m(2x) e^{-2x} [L_0(x) + \zeta^2 L_2(x)]^2, \quad (7)$$

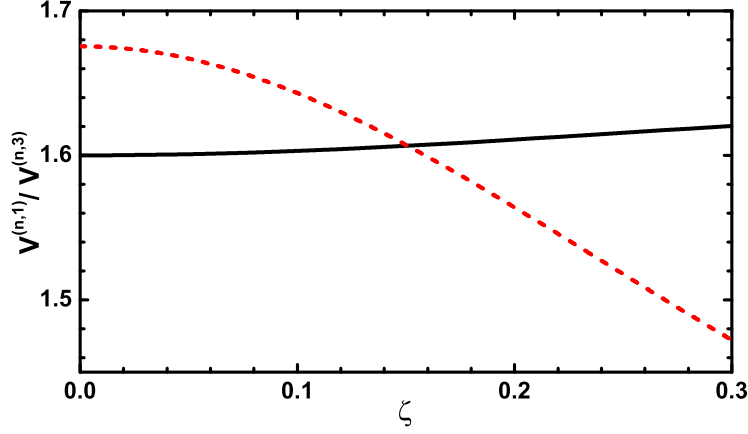


FIG. 2: (Color online) The ratio of the first and third relative angular momentum pseudopotentials for LLs $n=0$ (black solid line) and $n=\pm 1$ (red dashed line) as a function of the warping strength ζ .

and

$$V^{(\pm 1, m)} = \sum_{\mathbf{q}} \frac{\pi e^2}{\epsilon q} L_m(2x) e^{-2x} \left[\frac{1}{2} \zeta^2 x^3 + \frac{1}{4} (L_0(x) + L_1(x) + 6\zeta^2 L_3(x))^2 \right] \quad (8)$$

with $x \equiv \frac{q^2 l_B^2}{2}$ being a dimensionless variable. From Eq. (8) one can clearly see that the Haldane's pseudopotentials for LLs $n = 1$ and $n = -1$ are still identical in the presence of the warping term. This is different from the Zeeman splitting effect, which can induce an asymmetry in the pseudopotentials for $n=\pm 1$ LLs [17, 18].

Figure 1 plots the Haldane's pseudopotentials of Coulomb interaction $V^{(n, m)}$ for (a) $n=0$ and (b) $n=1$ LLs as functions of the relative angular momentum m . Comparing with the results in the absence of the warping term (black dots in Fig. 1), one can clearly find that in the presence of the warping term, the magnitude of the pseudopotentials increases (red stars in Fig. 1).

Subsequently, the stability of the TIFQH states should also be modified by the warping term. The typical results of $V^{(n,1)}/V^{(n,3)}$ ($n=0$ and $n=\pm 1$) are shown in Fig. 2 as a function of the warping parameter ζ . One can see that with increasing ζ , $V^{(0,1)}/V^{(0,3)}$ increases while $V^{(\pm 1,1)}/V^{(\pm 1,3)}$ decreases. According to the composite fermion theory, the larger the value of $V^{(n,1)}/V^{(n,3)}$ is, the more stable the fractional quantum Hall states. Therefore, the remarkable warping term results in the composite fermions at fractional filling for LL $n=0$

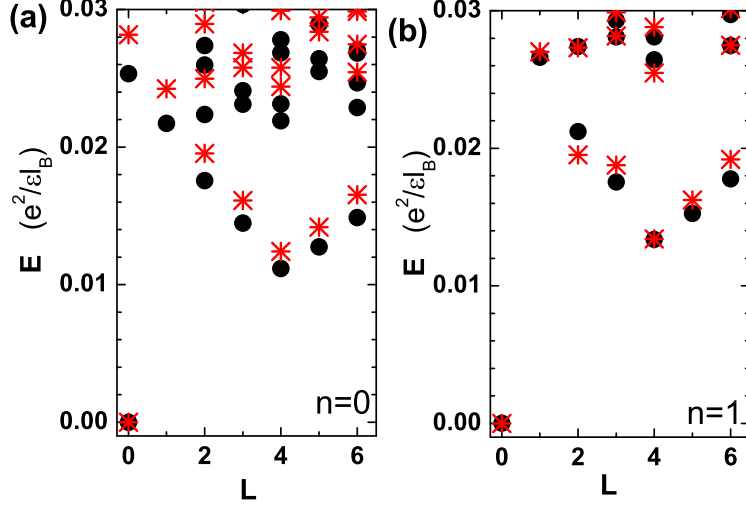


FIG. 3: (Color online) Exact energies versus the angular momentum L for $N=7$ electrons at $\nu=\frac{1}{3}$ TIFQH state. The warping strength is chosen as $\zeta=0$ (circles) and $\zeta=0.2$ (stars).

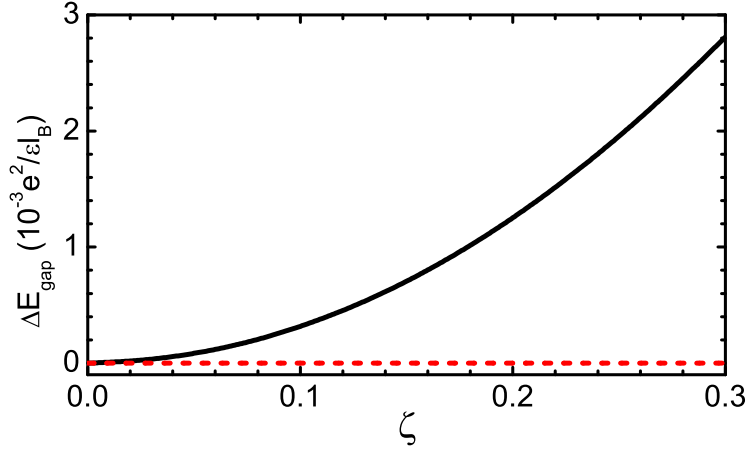


FIG. 4: (Color online) The increment of the gap width as a function of the warping term ζ for $N=7$ electrons at $\nu=\frac{1}{3}$ TIFQH states for LLs $n=0$ (black solid line) and $n=\pm 1$ (red dashed line).

(LLs $n=\pm 1$) to become more stable (unstable). This result suggests that on the surface of a TI material with strong (weak) warping strength, such as Bi_2Te_3 (Bi_2Se_3), the TIFQH states for LLs $n=0$ ($n=\pm 1$) may be observed much easier under a perpendicular magnetic field. We hope this prediction could be detected in future experiment.

In what follows, by using the exact diagonalization method in the spherical geometry, we investigate the system with the fractional filling factor $\nu=1/(2p+1)$, where p is an integer.

For briefness we only illustrate the $\nu=1/3$ TIFQH state, which is realized at $S = \frac{3}{2}(N - 1)$ in the spherical geometry with N being the electron number. Under this configuration, the perpendicular magnetic field is equivalent to a fictitious radial magnetic field produced by a magnetic monopole at the center of a sphere of radius $R = \sqrt{S}l_B$ (in unit of flux quanta), and the many-body states could be described by the total angular momentum L and its z component L_z .

We show in Fig. 3 the energy spectra of the many-body states at $\nu=1/3$ filling for $N=7$ electrons. Comparing the two cases with ($\zeta=0.2$) and without ($\zeta=0$) warping term, one can see from Fig. 3 that the gap width at $n=0$ LL between the ground state and the first excited state has a visible increment while those at $n=\pm 1$ LL keep unchanged. Furthermore, we calculate the corresponding excitation gap width $E_g^n(\zeta)$ by increasing the warping term from $\zeta=0$ to $\zeta=0.3$. The variation $\Delta E_g = E_g^n(\zeta) - E_g^n(\zeta = 0)$ as a function of ζ are plotted in Fig. 4, which shows that the TIFQH gap between the ground state and the lowest excited state at $n=0$ LL (solid line) are sensitively dependent on the warping term while ΔE_g at $n=\pm 1$ LLs (dashed line) keeps a constant no matter the warping term is included or not. This also implies that the warping term is different from the Zeeman splitting and the tilted magnetic field, which will induce a change in the gap of the TIFQH states at LLs $n=\pm 1$ [17, 18].

In summary, we perturbatively studied the effect of the warping term on the TIFQH states. It was found that differing from the role of the Zeeman splitting and the tilted magnetic field, the warping term does not break symmetry of the Haldane's pseudopotentials for $n=\pm 1$ LLs. Our results showed on one hand that, the stability of the $\nu = 1/3$ TIFQH states for LL $n = 0$ (LLs $n=\pm 1$) become stronger (weaker) by increasing the warping strength of the Fermi surface. On the other hand, the excitation gap for LL $n=0$ increases with increasing the strength of the warping term, while the gaps for LLs $n=\pm 1$ are yet insensitive to this term.

This work was supported by Natural Science Foundation of China under Grants No. 11274049, No. 10904005, No. 11004013, and by the National Basic Research Program of China (973 Program) under Grant No. 2009CB929103.

[1] M. Z. Hasan, and C. L. Kane, Rev. Mod. Phys., **82**, 3045 (2010).

- [2] X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011).
- [3] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science **314**, 1757 (2006).
- [4] L. Fu and C. L. Kane, Phys. Rev. B **76**, 045302 (2007).
- [5] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi and S.-C. Zhang, Science **318**, 766 (2007).
- [6] D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature **452**, 970 (2008).
- [7] H.-J. Zhang, C.-X. Liu, X.-L. Qi, X. Dai, Z. Fang, and S.-C. Zhang, Nat. Phys. **5**, 438 (2009).
- [8] Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, and R. J. Cava, Nat. Phys. **5**, 398 (2009).
- [9] Y.-L. Chen, J. G. Analytis, J.-H. Chu, Z.-K. Liu, S.-K. Mo, X.-L. Qi, H.-J. Zhang, D.-H. Lu, X. Dai, and Z. Fang, Science **325**, 178 (2009).
- [10] D. Hsieh *et al.*, Nature **460**, 1101 (2009).
- [11] L. Fu, Phys. Rev. Lett. **103**, 266801 (2009).
- [12] T. Zhang, P. Cheng, X. Chen, J.-F. Jia, X. Ma, K. He, L. Wang, H. Zhang, X. Dai, Z. Fang, X. Xie, and Q.-K. Xue, Phys. Rev. Lett. **103**, 266803 (2009).
- [13] J. Wang, W. Li, P. Cheng, C. Song, T. Zhang, P. Deng, X. Chen, X. Ma, K. He, J.-F. Jia, Q.-K. Xue, and B.-F. Zhu, Phys. Rev. B **84**, 235447 (2011).
- [14] J. Xue, J. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, P. Jarillo-Herrero, and B. J. LeRoy, Phys. Rev. Lett. **108**, 016801 (2012).
- [15] A. M. DaSilva, Solid State Communications **151**, 1444 (2011).
- [16] V. M. Apalkov and T. Chakraborty, Phys. Rev. Lett. **107**, 186801 (2011).
- [17] Z. Wang, F. Zheng, Z.-G. Fu, and P. Zhang, arXiv:1210.5137 (2012).
- [18] F. Zheng, Z. Wang, Z.-G. Fu, and P. Zhang, unpublished.
- [19] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983); F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **54**, 237 (1985).